Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about 0.5°.

Making Career Connections

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

Take Home Experiment: Determining g

Use a simple pendulum to determine the acceleration due to gravity g in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than 10°, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate g. How accurate is this measurement? How might it be improved?

CHECK YOUR UNDERSTANDING

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10 kg. Pendulum 2 has a bob with a mass of 100 kg. Describe how the motion of the pendula will differ if the bobs are both displaced by 12°.

Solution

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.

PHET EXPLORATIONS

Pendulum Lab

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of g on planet X. Notice the anharmonic behavior at large amplitude.

Click to view content (https://phet.colorado.edu/sims/pendulum-lab/pendulum-lab_en.html)

Figure 16.15

16.5 Energy and the Simple Harmonic Oscillator

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have We know from Hooke's Law: Stress and Strain Revisited that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:

$$\text{PE}_{\text{el}} = \frac{1}{2}kx^2. \tag{16.33}$$

Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy KE. Conservation of energy for these two forms is:

$$\mathrm{KE} + \mathrm{PE}_{\mathrm{el}} = \text{constant} \tag{16.34}$$

or

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}. \tag{16.35}$$

This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role

Namely, for a simple pendulum we replace the velocity with $v = L\omega$, the spring constant with $k = mg/L$, and the displacement term with $x = L\theta$. Thus

$$\frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 = \text{constant}. \tag{16.36}$$

In the case of undamped simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again in Figure 16.16, the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into elastic potential energy by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.

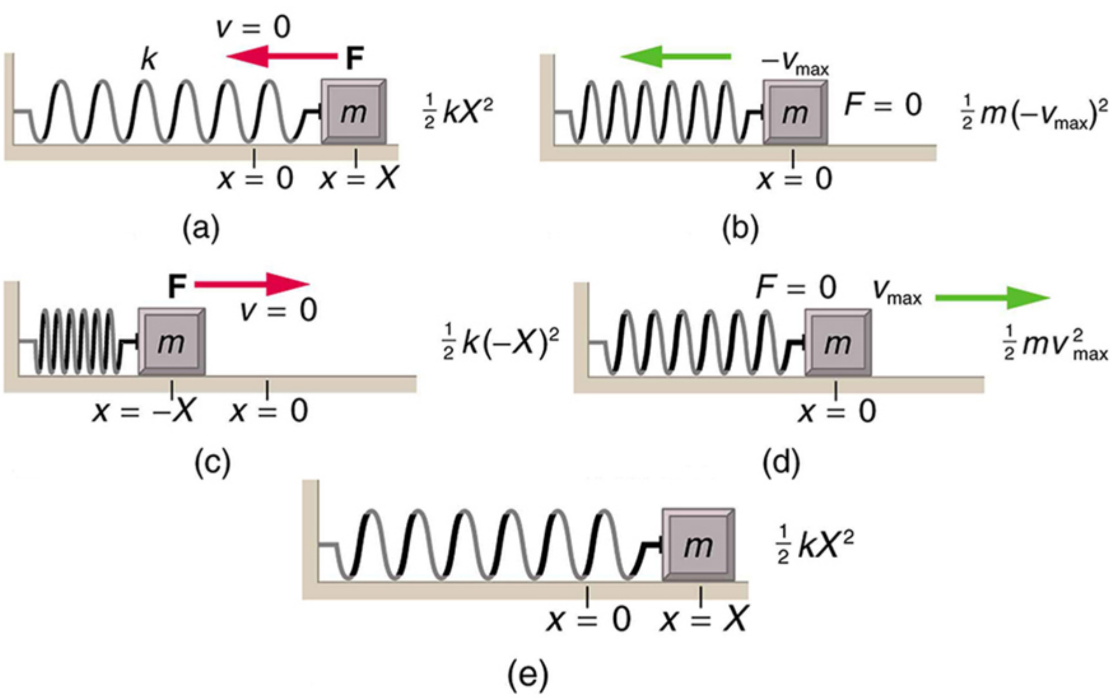

Figure 16.16 The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface.

The conservation of energy principle can be used to derive an expression for velocity v. If we start our simple harmonic motion with zero velocity and maximum displacement ($x = X$), then the total energy is

$$\frac{1}{2}kX^2. \tag{16.37}$$

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kX^2. \tag{16.38}$$

Solving this equation for v yields:

$$v = \pm\sqrt{\frac{k}{m}\left(X^2 - x^2\right)}. \tag{16.39}$$

Manipulating this expression algebraically gives:

$$v = \pm\sqrt{\frac{k}{m}}X\sqrt{1 - \frac{x^2}{X^2}} \tag{16.40}$$

and so

$$v = \pm v_{\max}\sqrt{1 - \frac{x^2}{X^2}}, \tag{16.41}$$

where

$$v_{\max} = \sqrt{\frac{k}{m}}X. \tag{16.42}$$

From this expression, we see that the velocity is a maximum ($v_{\max}$) at $x = 0$, as stated earlier in $v(t) = -v_{\max}\sin\frac{2\pi t}{T}$. Notice that the maximum velocity depends on three factors. Maximum velocity is directly proportional to amplitude. As you might guess, the greater the maximum displacement the greater the maximum velocity. Maximum velocity is also greater for stiffer systems, because they exert greater force for the same displacement. This observation is seen in the expression for $v_{\max}$; it is proportional to the square root of the force constant k. Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m. For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

$$\omega_{\max} = \sqrt{\frac{g}{L}}\theta_{\max}. \tag{16.43}$$

EXAMPLE 16.6

Determine the Maximum Speed of an Oscillating System: A Bumpy Road

Suppose that a car is 900 kg and has a suspension system that has a force constant $k = 6.53 \times 10^4$ N/m. The car hits a bump and bounces with an amplitude of 0.100 m. What is its maximum vertical velocity if you assume no damping occurs?

Strategy

We can use the expression for $v_{\max}$ given in $v_{\max} = \sqrt{\frac{k}{m}}X$ to determine the maximum vertical velocity. The variables m and k are given in the problem statement, and the maximum displacement X is 0.100 m.

Solution

1. Identify known.
2. Substitute known values into $v_{\max} = \sqrt{\frac{k}{m}}X$:

$$v_{\max} = \sqrt{\frac{6.53 \times 10^4\ \text{N/m}}{900\ \text{kg}}}(0.100\ \text{m}). \tag{16.44}$$

3. Calculate to find $v_{\max} = 0.852$ m/s.

Discussion

This answer seems reasonable for a bouncing car. There are other ways to use conservation of energy to find $v_{\max}$. We could use it directly, as was done in the example featured in Hooke's Law: Stress and Strain Revisited.

The small vertical displacement y of an oscillating simple pendulum, starting from its equilibrium position, is given as

$$y(t)= a \sin \omega t, \tag{16.45}$$

where a is the amplitude, ω is the angular velocity and t is the time taken. Substituting $\omega = \frac{2\pi}{T}$, we have

$$y(t) = a \sin \left(\frac{2\pi t}{T}\right). \tag{16.46}$$

Thus, the displacement of pendulum is a function of time as shown above.

Also the velocity of the pendulum is given by

$$v(t)= \frac{2a\pi}{T} \cos \left(\frac{2\pi t}{T}\right), \tag{16.47}$$

so the motion of the pendulum is a function of time.

CHECK YOUR UNDERSTANDING

Why does it hurt more if your hand is snapped with a ruler than with a loose spring, even if the displacement of each system is equal?

Solution

The ruler is a stiffer system, which carries greater force for the same amount of displacement. The ruler snaps your hand with greater force, which hurts more.

CHECK YOUR UNDERSTANDING

You are observing a simple harmonic oscillator. Identify one way you could decrease the maximum velocity of the system.

Solution

You could increase the mass of the object that is oscillating.

16.6 Uniform Circular Motion and Simple Harmonic Motion

Figure 16.17 The horses on this merry-go-round exhibit uniform circular motion. (credit: Wonderlane, Flickr)

There is an easy way to produce simple harmonic motion by using uniform circular motion. Figure 16.18 shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke's law usually describes uniform circular motions (ω constant) rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in Figure 16.18, is often easier than observing a precise large-scale simple harmonic oscillator. If studied in sufficient depth, simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.